Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2021

Mathematics: analysis and approaches

Higher level

Paper 3

23 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full* ***FT*** *is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g*. probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

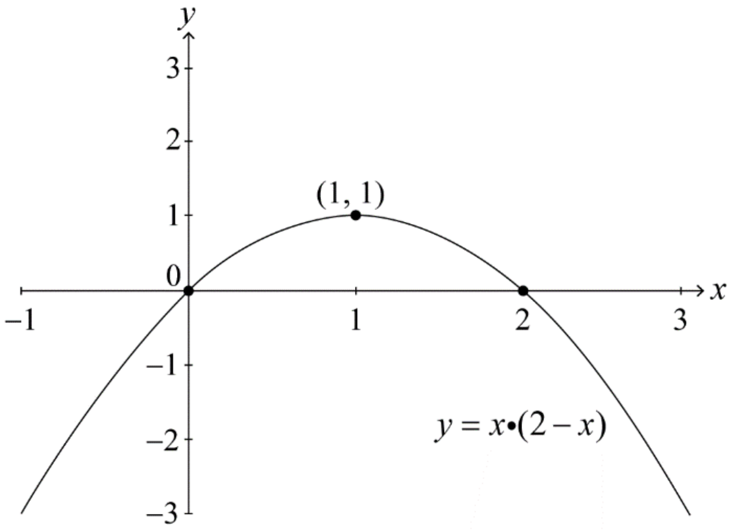
A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a)



inverted parabola extended below the x- axis ***A1***

x- axis intercept values $x = 0, 2$ ***A1***

Note: Accept a graph passing through the origin as an indication of $x = 0$.

local maximum at $(1,1)$ ***A1***

Note: Coordinates must be stated to gain the final ***A1***. Do not accept decimal approximations.

[3 marks]

(b)

	Number of local maximum points	**Number of local minimum points**	**Number of points of inflexion with zero gradient**
$n=3$ and $n=5$	1	0	2
$n=2$ and $n=4$	1	2	0

A1A1A1A1A1A1

Note: Award ***A1*** for each correct value.

For a table not sufficiently or clearly labelled, assume that their values are in the same order as the table in the question paper and award marks accordingly.

[6 marks]

(c) **METHOD 1**

attempts to use the product rule **(M1)**

$f_n'(x) = -nx^n(a-x)^{n-1} + nx^{n-1}(a-x)^n$ **A1A1**

Note: Award ***A1*** for a correct $u\frac{\mathrm{d}v}{\mathrm{d}x}$ and ***A1*** for a correct $v\frac{\mathrm{d}u}{\mathrm{d}x}$.

EITHER

attempts to factorise $f_n'(x)$ (involving at least one of nx^{n-1} or $(a-x)^{n-1}$) **(M1)**

$= nx^{n-1}(a-x)^{n-1}((a-x)-x)$ **A1**

OR

attempts to express $f_n'(x)$ as the difference of two products with each product containing at least one of nx^{n-1} or $(a-x)^{n-1}$ **(M1)**

$= (-x)(nx^{n-1})(a-x)^{n-1} + (a-x)(nx^{n-1})(a-x)^{n-1}$ **A1**

THEN

$f_n'(x) = nx^{n-1}(a-2x)(a-x)^{n-1}$ **AG**

Note: Award the final ***(M1)A1*** for obtaining any of the following forms:

$$f_n'(x) = nx^n(a-x)^n\left(\frac{a-x-x}{x(a-x)}\right); \quad f_n'(x) = \frac{nx^n(a-x)^n}{x(a-x)}(a-x-x);$$

$$f_n'(x) = nx^{n-1}\left((a-x)^n - x(a-x)^{n-1}\right);$$

$$f_n'(x) = (a-x)^{n-1}\left(nx^{n-1}(a-x)^n - nx^n\right)$$

METHOD 2

$f_n(x) = (x(a-x))^n$ **(M1)**

$= (ax-x^2)^n$ **A1**

attempts to use the chain rule **(M1)**

$f_n'(x) = n(a-2x)(ax-x^2)^{n-1}$ **A1A1**

Note: Award ***A1*** for $n(a-2x)$ and ***A1*** for $\left(ax-x^2\right)^{n-1}$.

$f_n'(x)=nx^{n-1}(a-2x)(a-x)^{n-1}$ ***AG***

[5 marks]

(d) $x=0,\ x=\frac{a}{2},\ x=a$ ***A2***

Note: Award ***A1*** for either two correct solutions or for obtaining $x=0,\ x=-a,\ x=-\frac{a}{2}$. Award ***A0*** otherwise.

[2 marks]

(e) attempts to find an expression for $f_n\left(\frac{a}{2}\right)$ ***(M1)***

$f_n\left(\frac{a}{2}\right)=\left(\frac{a}{2}\right)^n\left(a-\frac{a}{2}\right)^n$

$=\left(\frac{a}{2}\right)^n\left(\frac{a}{2}\right)^n\ \left(=\left(\frac{a}{2}\right)^{2n}\right),\left(=\left(\left(\frac{a}{2}\right)^n\right)^2\right)$ ***A1***

EITHER

since $a\in\mathbb{R}^+$, $\left(\frac{a}{2}\right)^{2n}>0$ (for $n\in\mathbb{Z}^+,n>1$ and so $f_n\left(\frac{a}{2}\right)>0$) ***R1***

Note: Accept any logically equivalent conditions/statements on a and n. Award ***R0*** if any conditions/statements specified involving a, n or both are incorrect.

OR

(since $a\in\mathbb{R}^+$), $\frac{a}{2}$ raised to an even power ($2n$) (or equivalent reasoning) is always positive (and so $f_n\left(\frac{a}{2}\right)>0$) ***R1***

Note: The condition $a\in\mathbb{R}^+$ is given in the question. Hence some candidates will assume $a\in\mathbb{R}^+$ and not state it. In these instances, award ***R1*** for a convincing argument. Accept any logically equivalent conditions/statements on a and n. Award ***R0*** if any conditions/statements specified involving a, n or both are incorrect.

THEN

so $\left(\frac{a}{2}, f_n\left(\frac{a}{2}\right)\right)$ is always above the horizontal axis **AG**

Note: Do not award ***(M1)A0R1***.

[3 marks]

(f) **METHOD 1**

$f_n'\left(\frac{a}{4}\right)=n\left(\frac{a}{4}\right)^{n-1}\left(a-\frac{a}{2}\right)\left(a-\frac{a}{4}\right)^{n-1}\ \left(=n\left(\frac{a}{4}\right)^{n-1}\left(\frac{a}{2}\right)\left(\frac{3a}{4}\right)^{n-1}\right)$ **A1**

EITHER

$n\left(\frac{a}{4}\right)^{n-1}\left(\frac{a}{2}\right)\left(\frac{3a}{4}\right)^{n-1}>0$ as $a\in\mathbb{R}^+$ and $n\in\mathbb{Z}^+$ **R1**

OR

$n\left(\frac{a}{4}\right)^{n-1}, \left(a-\frac{a}{2}\right)$ and $\left(a-\frac{a}{4}\right)^{n-1}$ are all >0 **R1**

Note: Do not award ***A0R1***.

Accept equivalent reasoning on correct alternative expressions for $f_n'\left(\frac{a}{4}\right)$ and accept any logically equivalent conditions/statements on a and n.

Exceptions to the above are condone $n>1$ and condone $n>0$.

An alternative form for $f_n'\left(\frac{a}{4}\right)$ is $(2n)(3)^{n-1}\left(\frac{a}{4}\right)^{2n-1}$.

THEN

hence $f_n'\left(\frac{a}{4}\right)>0$ **AG**

[2 marks]

METHOD 2

$f_n(0)=0$ and $f_n\left(\frac{a}{2}\right)>0$ **A1** (since f_n is continuous and there are no stationary points between $x=0$ and $x=\frac{a}{2}$)

the gradient (of the curve) must be positive between $x=0$ and $x=\frac{a}{2}$ ***R1***

Note: Do not award ***A0R1***.

hence $f_n'\left(\frac{a}{4}\right)>0$ ***AG***

[2 marks]

(g) (i) $f_n'(-1)=n(-1)^{n-1}(a+2)(a+1)^{n-1}$

for n even:

$n(-1)^{n-1}(=-n)<0$ (and $(a+2),(a+1)^{n-1}$ are both >0) ***R1***

$f_n'(-1)<0$ ***A1***

$f_n'(0)=0$ and $f_n'\left(\frac{a}{4}\right)>0$ (seen anywhere) ***A1***

Note: Candidates can give arguments based on the sign of $(-1)^{n-1}$ to obtain the ***R*** mark.

For example, award ***R1*** for the following:

If n is even, then $n-1$ is odd and hence $(-1)^{n-1}<0$ $(=-1)$.

Do not award ***R0A1***.

The second ***A1*** is independent of the other two marks.

The ***A*** marks can be awarded for correct descriptions expressed in words.

Candidates can state $(0,0)$ as a point of zero gradient from part (d) or show, state or explain (words or diagram) that $f_n'(0)=0$. The last ***A*** mark can be awarded for a clearly labelled diagram showing changes in the sign of the gradient.

The last ***A1*** can be awarded for use of a specific case (*e.g.* $n=2$).

hence $(0,0)$ is a local minimum point ***AG***

[3 marks]

(ii) for n odd:

$n(-1)^{n-1}(=n)>0$, (and $(a+2)$, $(a+1)^{n-1}$ are both >0) so $f_n'(-1)>0$ ***R1***

Note: Candidates can give arguments based on the sign of $(-1)^{n-1}$ to obtain the ***R*** mark.

For example, award ***R1*** for the following:

If n is odd, then $n-1$ is even and hence $(-1)^{n-1}>0$ $(=1)$.

$f_n'(0)=0$ and $f_n'\left(\frac{a}{4}\right)>0$ (seen anywhere) ***A1***

Note: The ***A1*** is independent of the ***R1***.

Candidates can state $(0,0)$ as a point of zero gradient from part (d) or show, state or explain (words or diagram) that $f_n'(0)=0$. The last *A* mark can be awarded for a clearly labelled diagram showing changes in the sign of the gradient.

The last *A1* can be awarded for use of a specific case (*e.g.* $n=3$).

hence $(0,0)$ is a point of inflexion with zero gradient ***AG***

[2 marks]

(h) considers the parity of n ***(M1)***

Note: Award ***M1*** for stating at least one specific even value of n.

n must be even (for four solutions) ***A1***

Note: The above 2 marks are independent of the 3 marks below.

$$0 < k < \left(\frac{a}{2}\right)^{2n}$$ ***A1A1A1***

Note: Award ***A1*** for the correct lower endpoint, ***A1*** for the correct upper endpoint and ***A1*** for strict inequality signs.

The third ***A1*** (strict inequality signs) can only be awarded if ***A1A1*** has been awarded.

For example, award ***A1A1A0*** for $0 \le k \le \left(\frac{a}{2}\right)^{2n}$. Award ***A1A0A0*** for $k > 0$.

Award ***A1A0A0*** for $0 < k < f_n\left(\frac{a}{2}\right)$.

[5 marks]

Total [31 marks]

2. (a) (i) **METHOD 1**

attempts to expand $\left(\omega-1\right)\left(\omega^2+\omega+1\right)$ ***(M1)***

$=\omega^3+\omega^2+\omega-\omega^2-\omega-1$ ***A1***

$=\omega^3-1$ ***AG***

[2 marks]

METHOD 2

attempts polynomial division on $\dfrac{\omega^3-1}{\omega-1}$ ***M1***

$=\omega^2+\omega+1$ ***A1***

so $\left(\omega-1\right)\left(\omega^2+\omega+1\right)=\omega^3-1$ ***AG***

[2 marks]

(ii) (since ω is a root of $z^3=1)\Rightarrow\omega^3-1=0$ ***R1***

and $\omega\neq 1$ ***R1***

$\Rightarrow\omega^2+\omega+1=0$ ***AG***

Note: In part (a), award marks as appropriate where ω has been converted into Cartesian, modulus-argument (polar) or Euler form.

[2 marks]

(b) **METHOD 1**

attempts to find either P_0P_1 or P_0P_2 ***(M1)***

accept any valid method

e.g. $2\sin\frac{\pi}{3}$, $1^2+1^2-2\cos\frac{2\pi}{3}$, $\frac{1}{\sin\frac{\pi}{6}}=\frac{P_0P_1}{\sin\frac{2\pi}{3}}$ from either ΔOP_0P_1 or ΔOP_0P_2

e.g. use of Pythagoras' theorem

e.g. $\left|1-e^{i\frac{2\pi}{3}}\right|$, $\left|1-\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\right|$ by calculating the distance between 2 points

$P_0P_1=\sqrt{3}$ ***A1***

$P_0P_2=\sqrt{3}$ ***A1***

Note: Award a maximum of ***M1A1A0*** for any decimal approximation seen in the calculation of either P_0P_1 or P_0P_2 or both.

so $P_0P_1\times P_0P_2=3$ ***AG***

METHOD 2

attempts to find $P_0P_1\times P_0P_2=|1-\omega||1-\omega^2|$ ***(M1)***

$P_0P_1\times P_0P_2=|\omega^3-\omega^2-\omega+1|$ ***A1***

$=|1-(\omega^2+\omega+1)+2|$ and since $\omega^2+\omega+1=0$ ***R1***

so $P_0P_1\times P_0P_2=3$ ***AG***

[3 marks]

(c) **METHOD 1**

$z^4 - 1 = (z-1)(z^3 + z^2 + z + 1)$ ***A1***

(ω is a root hence) $\omega^4 - 1 = 0$ and $\omega \neq 1$ ***R1***

$\Rightarrow \omega^3 + \omega^2 + \omega + 1 = 0$ ***AG***

Note: Condone the use of ω throughout.

[2 marks]

METHOD 2

considers the sum of roots of $z^4 - 1 = 0$ ***(M1)***

the sum of roots is zero (there is no z^3 term) ***A1***

$\Rightarrow \omega^3 + \omega^2 + \omega + 1 = 0$ ***AG***

[2 marks]

METHOD 3

substitutes for ω ***(M1)***

e.g. LHS $= e^{i\frac{3\pi}{2}} + e^{\pi i} + e^{i\frac{\pi}{2}} + 1$

$= -i - 1 + i + 1$ ***A1***

Note: This can be demonstrated geometrically or by using vectors. Accept Cartesian or modulus-argument (polar) form.

$\Rightarrow \omega^3 + \omega^2 + \omega + 1 = 0$ ***AG***

[2 marks]

METHOD 4

$\omega^3 + \omega^2 + \omega + 1 = \dfrac{\omega^4 - 1}{\omega - 1}$ ***A1***

$= \dfrac{0}{\omega - 1} = 0$ as $\omega \neq 1$ ***R1***

$\Rightarrow \omega^3 + \omega^2 + \omega + 1 = 0$ ***AG***

[2 marks]

(d) **METHOD 1**

$P_0P_2 = 2$ ***A1***

attempts to find either P_0P_1 or P_0P_3 ***(M1)***

Note: For example, $P_0P_1 = \lvert 1-\mathrm{i}\rvert$ and $P_0P_3 = \lvert 1+\mathrm{i}\rvert$. Various geometric and trigonometric approaches can be used by candidates.

$P_0P_1 = \sqrt{2}$, $P_0P_3 = \sqrt{2}$ ***A1A1***

Note: Award a maximum of ***A1M1A1A0*** if labels such as P_0P_1 are not clearly shown. Award full marks if the lengths are shown on a clearly labelled diagram. Award a maximum of ***A1M1A1A0*** for any decimal approximation seen in the calculation of either P_0P_1 or P_0P_3 or both.

$P_0P_1 \times P_0P_2 \times P_0P_3 = 4$ ***AG***

[4 marks]

METHOD 2

attempts to find $P_0P_1 \times P_0P_2 \times P_0P_3 = \lvert 1-\omega\rvert \lvert 1-\omega^2\rvert \lvert 1-\omega^3\rvert$ ***M1***

$P_0P_1 \times P_0P_2 \times P_0P_3 = \lvert -\omega^6 + \omega^5 + \omega^4 - \omega^2 - \omega + 1\rvert$ ***A1***

$= \lvert -(-1) + \omega^5 + 1 - (-1) - \omega + 1\rvert$ since $\omega^6 = \omega^2 = -1$ and $\omega^4 = 1$ ***A1***

$= \lvert \omega^5 - \omega + 4\rvert$ and since $\omega^5 = \omega$ ***R1***

so $P_0P_1 \times P_0P_2 \times P_0P_3 = 4$ ***AG***

[4 marks]

METHOD 3

$P_0P_2 = 2$ ***A1***

attempts to find $P_0P_1 \times P_0P_3 = |1-\omega||1-\omega^3|$ ***M1***

$P_0P_1 \times P_0P_3 = |\omega^4 - \omega^3 - \omega + 1|$ ***A1***

$= |2-(-\omega)-\omega|$ since $\omega^4 = 1$ and $\omega^3 = -\omega$ ***R1***

so $P_0P_1 \times P_0P_2 \times P_0P_3 = 4$ ***AG***

[4 marks]

(e) $\left(\mathrm{P_0P_1 \times P_0P_2 \times ... \times P_0P_{n\text{-}1}}\right) = n$ ***A1***

[1 mark]

(f) (i) $\mathrm{P_0P_2} = \left|1-\omega^2\right|,\ \mathrm{P_0P_3} = \left|1-\omega^3\right|$ ***A1A1***

[2 marks]

(ii) $\mathrm{P_0P}_{n-1} = \left|1-\omega^{n-1}\right|$ ***A1***

Note: Accept $\left|1-\omega\right|$ from symmetry.

[1 mark]

(g) (i) $z^n - 1 = (z-1)(z^{n-1} + z^{n-2} + ... + z + 1)$

considers the equation $z^{n-1} + z^{n-2} + ... + z + 1 = 0$ ***(M1)***

the roots are $\omega, \omega^2, ..., \omega^{n-1}$ ***(A1)***

so $(z-\omega)(z-\omega^2)...(z-\omega^{n-1})$ ***A1***

[3 marks]

(ii) **METHOD 1**

substitutes $z=1$ into $(z-\omega)(z-\omega^2)...(z-\omega^{n-1}) \equiv z^{n-1} + z^{n-2} + ... + z + 1$

M1

$(1-\omega)(1-\omega^2)...(1-\omega^{n-1}) = n$ ***(A1)***

takes modulus of both sides ***M1***

$\left|(1-\omega)(1-\omega^2)...(1-\omega^{n-1})\right| = |n|$

$|1-\omega||1-\omega^2|...|1-\omega^{n-1}| = n$ ***A1***

so $\mathrm{P_0P_1} \times \mathrm{P_0P_2} \times ... \times \mathrm{P_0P}_{n-1} = n$ ***AG***

Note: Award a maximum of ***M1A1FTM1A0*** from part (e).

[4 marks]

METHOD 2

$(1-\omega),(1-\omega^2),\ldots,(1-\omega^{n-1})$ are the roots of $(1-v)^{n-1}+(1-v)^{n-2}+\ldots+(1-v)+1=0$ ***M1***

coefficient of v^{n-1} is $(-1)^{n-1}$ and the coefficient of 1 is n ***A1***

product of the roots is $\frac{(-1)^{n-1}n}{(-1)^{n-1}}=n$ ***A1***

$|1-\omega||1-\omega^2|\ldots|1-\omega^{n-1}|=n$ ***A1***

so $P_0P_1\times P_0P_2\times\ldots\times P_0P_{n-1}=n$ ***AG***

[4 marks]

Total[24 marks]